## CUHK-CUNY-2023

## Compactness and Scalar Curvature Workshop

**An Online Workshop organized by**
Man Chun Lee (CUHK)
Christina Sormani (CUNY)

**Description**: In this workshop we will consider sequences of three dimensional Riemannian manifolds with various scalar curvature bounds. Speakers will present their results concerning such sequences, reviewing relevant prior research and outstanding conjectures, and carefully explaining the notions of convergence they apply. All are welcome to join this online workshop via zoom.

**Dates and Times**
NYC evenings 9pm-12midnight July 10-13
California evenings 6-9 pm July 10-13
Hong Kong mornings 9am-12noon July 11-14

**Schedule:**
Each day we have two one hour talks with half hour discussions after the talk.
The 1st talk is at 9:00 am in Hong Kong which is 9:00 pm in New York.
The 2nd talk is at 10:30 am in Hong Kong which is 10:30 pm in New York.

**Day 1**
**Shaodong Wang**
*Compactness Theorems for conformal metrics with constant scalar curvature in dimension three*
Discussion - Brian Allen
**Wenchuan Tian**
*An Extreme Example related to the Compactness Conjecture for Scalar ge 0 and MinA ge A*
Discussion - Jiewon Park

**Day 2**
**Raquel Perales**
*Volume Above Distance Below, Almost Rigidity of Tori, and Intrinsic Flat Convergence*
Discussion - Christina Sormani
**Changliang Wang**
*Scalar MinA Compactness for Warped Product Manifolds*
Discussion - Wenchuan Tian

**Day 3**
**Brian Allen**
*Almost Rigidity of the Llarull Thm*
Discussion - Edward Bryden

**Jian Wang**
*Topology of complete $3$-manifolds with uniformly positive scalar curvature.*
Discussion - Changliang Wang

**Day 4**
**Jintian Zhu**
*The Gauss-Bonnet inequality beyond aspherical conjecture*
Discussion - Man-Chun Lee
**Kai Xu**
*A Topological Gap Theorem and Inverse Mean Curvature Flow*
Discussion - Christina Sormani

**Zoom Info**
Join Zoom Meeting
https://cuhk.zoom.us/j/95988240027?pwd=WFIxZ216Z0tEQ0NxTlBhMjhuY1FZZz09
Meeting ID: 959 8824 0027
Passcode is six numbers: five-four-nine-seven-zero-eight

## Abstracts:

**Shaodong Wang (Day 1 Talk 1)**
***Compactness Theorems for conformal metrics with constant scalar curvature in dimension three***

In this talk, I will be presenting results on the compactness of Yamabe problems on general three-dimensional Riemannian manifolds with boundaries. The Yamabe problem is a classical topic of study in conformal geometry that concerns the existence of metrics with constant curvatures. In my presentation, I will be focusing on the cases with constant scalar curvature in the interior and constant boundary mean curvature as well. This involves a blow-up argument for nonlinear partial differential equations with critical nonlinearities both in the interior and on the boundary. The talk is based on joint work with Sergio Almaraz (https://arxiv.org/abs/2306.07088) and with both S. Almaraz and Olivaine Queiroz (https://arxiv.org/abs/1807.08406).

**Wenchuan Tian (Day 1 Talk 2)**
***An Extreme Example related to the Compactness Conjecture for Scalar ge 0 and MinA ge A***

In 2014, Gromov vaguely conjectured that a sequence of manifolds with nonnegative scalar curvature should have a subsequence which converges in some weak sense to a limit space with some generalized notion of nonnegative scalar curvature. The conjecture has been made precise at an IAS Emerging Topics meeting: requiring that the sequence be three dimensional with uniform upper bounds on diameter and volume, and a positive uniform lower bound on MinA, which is the minimum area of a closed minimal surface in the manifold (https://arxiv.org/abs/2103.10093). Here we present a sequence of warped product manifolds with warped circles over standard spheres, that have circular fibres over the poles whose length diverges to infinity, that satisfy the hypotheses of this IAS conjecture. We prove this sequence converges in the W1,p sense for p<2 an extreme limit space that has nonnegative scalar curvature in the distributional sense as defined by Lee-LeFloch and that the total distributional scalar curvature converges. The sequences also converges in the uniform, Gromov-Hausdorff, and Intrinsic Flat sense to the metric completion of the extreme limit space which is homeomorphic to a product of a standard two sphere with a circle. This is joint work with Changliang Wang and Christina Sormani (https://arxiv.org/abs/2304.07000) and a second paper to appear with C. Sormani.

**Raquel Perales (Day 2 Talk 1)**
***Volume Above Distance Below, Almost Rigidity of Tori, and Intrinsic Flat Convergence***

Given a pair of metric tensors gj≥g0 on a Riemannian manifold, M, it is well known that Volj(M)≥Vol0(M). Furthermore, the volumes are equal if and only if the metric tensors are the same gj=g0. Here we prove that if for a sequence gj, we have gj≥g0, Volj(M)→Vol0(M) and diam(Mj) \leq D then (M,gj) converges to (M,g0) in the volume preserving intrinsic flat sense. This theorem will then be applied to prove stability of a class of tori. This talk is be based
on joint work of myself with: Allen and Sormani (https://arxiv.org/abs/2003.01172), and Cabrera Pacheco and Ketterer (https://arxiv.org/abs/1902.03458).

**Changliang Wang (Day 2 Talk 2)**
***Scalar MinA Compactness for Warped Product Manifolds***

Gromov and Sormani conjectured that a sequence of three dimensional Riemannian manifolds with nonnegative scalar curvature and some additional uniform geometric bounds should have a subsequence which converges in some sense to a limit space with generalized notion of nonnegative scalar curvature. In this paper, we study the precompactness of a sequence of three dimensional warped product manifolds with warped circles over standard spheres that have nonnegative scalar curvature, a uniform upper bound on the volume, and a positive uni-
form lower bound on the MinA, which is the minimum area of closed minimal surfaces in the manifold. We prove that such a sequence has a subsequence converging to a W1,p Riemannian metric for all p < 2, and that the limit metric has nonnegative scalar curvature in the distri-
butional sense as defined by Lee-LeFloch. This is joint work with Wenchuan Tian (arxiv to appear this week). See also previous work with W. Tian and Jiewon Park (https://arxiv.org/abs/1812.03502).

**Brian Allen (Day 3 Talk 1)**
***Almost Rigidity of the Llarull Thm***

Llarull's Theorem states that any Riemannian metric on the $n$-sphere which has scalar curvature greater than or equal to $n(n-1)$, and whose distance function is bounded below by the unit sphere's, is isometric to the unit sphere. Gromov later posed the Spherical Stability Problem, probing the flexibility of this fact, which we give a resolution of in dimension $3$. We show that a sequence of Riemannian $3$-spheres whose distance functions are bounded below by the unit sphere's with uniformly bounded Cheeger isoperimetric constant and scalar curvatures tending to $6$ must approach the round $3$-sphere in the volume preserving Sormani-Wenger Intrinsic Flat sense. The argument is based on a proof of Llarull's Theorem due to Hirsch-Kazaras-Khuri-Zhang (https://arxiv.org/abs/2209.12857) using spacetime harmonic functions and a characterization of Sormani-Wenger Intrinsic Flat convergence given by Allen-Perales-Sormani (https://arxiv.org/abs/2003.01172). This is joint work with E. Bryden and D. Kazaras (https://arxiv.org/abs/2305.18567).

**Jian Wang (Day 3 Talk 2)**
***Topology of complete $3$-manifolds with uniformly positive scalar curvature.***

Abstract: A well-known question posed by S.T. Yau is how to classify 3-manifolds admitting a complete metric with (uniformly) positive scalar curvature up to diffeomorphism. It was resolved by G. Perelman for closed $3$-manifolds. However, the non-compact case is complicated. In this talk, I will give a complete topological characterization for complete open $3$-manifolds with uniformly positive scalar curvature. Furthermore, we will talk about its generalization for $3$-manifolds with boundaries. See https://arxiv.org/abs/2212.14383.

**Jintian Zhu (Day 4 Talk 1)**
***The Gauss-Bonnet inequality beyond aspherical conjecture***

In this talk, I plan to first review the topic of largeness behind the aspherical conjecture, and then focus on the following Gauss-Bonnet inequality beyond the aspherical conjecture: if the universal covering of a closed manifold with nonnegative scalar curvature has ``homological dimension no greater than two'', then either it is flat or its Gauss-Bonnet quantity is no greater than $8\pi$, where the Gauss-Bonnet quantity is the infimum of ambient-scalar-curvature-integral over homotopically non-trivial two-spheres. I'll also mention my conjecture on Gauss-Bonnet inequality with boundary following the view of geometry-over-topology principle. See https://arxiv.org/abs/2206.07955.

**Kai Xu (Day 4 Talk 2)**
***A Topological Gap Theorem and Inverse Mean Curvature Flow***

Given a closed orientable 3-manifold M with scalar curvature greater or equal than 1, it is well-known that if $\pi_2(M)\neq 0$ then the $\pi_2$-systole of M is at most $8\pi$ . We prove the following gap theorem: if M is further not a quotient of $S^2\times S^1$, then the $\pi_2$-systole of M is no greater than an improved constant $c\approx 5.44\pi$. This statement follows as a new application of Huisken and Ilmanen's weak inverse mean curvature flow. See https://arxiv.org/abs/2307.01922.

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