

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520C University Mathematics for Applications 2014-2015
Test 1, 12 Feb, 2015

- Time allowed: 45 minutes
- Answer all questions.
- Show your work clearly and concisely in your answer book.
- Write down your name and student ID number on the front page of your answer book.
- You are allowed to use a calculator in this test.

1. Evaluate each of the following limits.

(a) $\lim_{x \rightarrow 5} \frac{x^3 - 125}{x - 5}$.

(b) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$.

(14 points)

2. Evaluate each of the following limits.

(a) $\lim_{x \rightarrow +\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$.

(b) $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 - 4x}}$.

(16 points)

3. Let $f(x) = x|x|$, where x is a real number.

(a) Prove that f is continuous at $x = 0$.

(b) Prove that f is differentiable at $x = 0$ and find $f'(0)$.

(18 points)

4. Find the derivative $f'(x)$ if $f(x) = \frac{e^{5x} \sqrt[4]{2x-5}}{(3x-5)^3}$

(12 points)

5. A toxin is introduced into a bacterial colony, and t hours later, the population is estimated by the function

$$N(t) = 5000(2 + t)e^{-0.01t}.$$

- (a) What is the population when the toxin is introduced?
- (b) When is the population maximized? What is the maximum population at that time?
- (c) What happens to the population in the long run?

(20 points)

6. Let $f(x) = x^2 + \frac{1}{x-1}$, where x is a real number and $x \neq 1$.

- (a) Find $f'(x)$ and $f''(x)$ for $x \neq 1$.
- (b) Find the range of x such that
 - (i) $f'(x) > 0$
 - (ii) $f'(x) < 0$
 - (iii) $f''(x) > 0$
 - (iv) $f''(x) < 0$
- (c) Find the local maximum and minimum points and the points of inflection, if any.
- (d) Find horizontal and vertical asymptotes of the graph of $f(x)$, if any.
- (e) Sketch the graph of $f(x)$.

(20 points)