

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520C University Mathematics for Applications 2015
Suggested Solution to Assignment 1

Chapter1.5:23 Ans: $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^2 + 3x + 2} = \lim_{x \rightarrow -2} \frac{(x-3)(x+2)}{(x+2)(x+1)} = \lim_{x \rightarrow -2} \frac{x-3}{x+1} = \frac{-5}{-1} = 5$

Chapter1.5:25 Ans: $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{(\sqrt{x} - 2)(\sqrt{x} + 2)} = \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$

Chapter1.5:59 Ans:

a. $C(0) = 0.4 + 0.013 = 0.413$ mg/ml

b. $C(5) - C(4) = \frac{0.4}{5^{1.2} + 1} - \frac{0.4}{4^{1.2} + 1} \approx -0.013$. Therefore the concentration decreases by about 0.013 mg/ml.

c. $\lim_{t \rightarrow \infty} C(t) = 0.013$. Therefore the residual concentration is 0.013 mg/ml.

Chapter1.6:27 Ans: By definition $f(3) = 10$. And

$$\lim_{x \rightarrow 3^-} f(x) = 10 \quad \lim_{x \rightarrow 3^+} f(x) = 10$$

$$\implies \lim_{x \rightarrow 3} f(x) = 10 = f(3). \text{ Therefore } f(x) \text{ is continuous at } x = 3.$$

Chapter1.6:57 Ans: By the definition $f(2) = 9$ and $\lim_{x \rightarrow 2^-} f(x) = 2A - 3 = f(2)$ since f is continuous at $x = 2$.

$$\implies A = 6$$

1 Ans: $\lim_{x \rightarrow +\infty} \frac{e^x - 2}{e^{2x} - 4} = \lim_{x \rightarrow +\infty} \frac{\frac{e^x - 2}{e^{2x}}}{\frac{e^{2x} - 4}{e^{2x}}} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{e^x} - \frac{2}{e^{2x}}}{1 - \frac{4}{e^{2x}}} = \frac{0 + 0}{1 - 0} = 0$

2 Ans:

(a) By definition, $f(0) = 0$

(b) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 1 = 1$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} e^{2x} = 1$$

Since $\lim_{x \rightarrow 0^-} f(x) = 1 = \lim_{x \rightarrow 0^+} f(x)$, we have $\lim_{x \rightarrow 0} f(x) = 1$

(c) Since $\lim_{x \rightarrow 0} f(x) = 1 \neq f(0)$, $f(x)$ is not continuous at $x = 0$.