MATH1010 University Mathematics
Supplementary Exercise

Chapter 1: Functions

1. For each of the following functions, find the maximum domain of definition of the function and the range of the function with this domain.

   (a) $f(x) = x^3 - 3x + 5$

   (b) $f(x) = \sqrt{7 - 2x}$

   (c) $f(x) = \dfrac{x+4}{x^2 - 3x - 10}$

   (d) $f(x) = \dfrac{\sqrt{x}}{x^2 + 2x + 5}$

   (e) $f(x) = \dfrac{1}{\sqrt{x^2 - 4}}$

   (f) $f(x) = \dfrac{1}{\sin x}$

   (g) $f(x) = \dfrac{1}{\sin x + \cos x}$

   (h) $f(x) = \ln(x - 3)$

   (i) $f(x) = \dfrac{2}{1 - \ln x}$

   (j) $f(x) = \sqrt{3 + \ln x}$

   (k) $f(x) = \ln(\ln x)$

   (l) $f(x) = \sqrt{2 - |\ln(1 - x)|}$

2. For each of the following functions, determine whether it is injective, surjective or bijective.

   (a) $f : \mathbb{R} \to \mathbb{R};\ f(x) = x^3$

   (b) $f : \mathbb{R} \to \mathbb{R}^+;\ f(x) = \dfrac{1}{x^2}$

   (c) $f : \mathbb{R}^+ \to \mathbb{R};\ f(x) = \ln x$

   (d) $f : \mathbb{R}^+ \to \mathbb{R}^+;\ f(x) = |x - 2| + 3$

   (e) $f : \mathbb{R}\backslash\{2\} \to \mathbb{R};\ f(x) = \dfrac{3x+1}{x-2}$

   (f) $f : \mathbb{R} \to \mathbb{R};\ f(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

   (g) $f : \mathbb{R} \to \mathbb{R};\ f(x) = \dfrac{e^x - e^{-x}}{2}$

   (h) $f : \mathbb{R} \to \mathbb{R};\ f(x) = \ln(x + \sqrt{x^2 + 1})$

3. Sketch the graphs of the following functions.

   (a) $f(x) = x^2 - 4x - 5$

   (b) $f(x) = \dfrac{2x - 9}{x + 3},\ x \neq -3$

   (c) $f(x) = \dfrac{x^2}{x - 2},\ x \neq 2$

   (d) $f(x) = 3 - \sqrt{4 - x^2},\ -2 \leq x \leq 2$

   (e) $f(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

   (f) $f(x) = 3 - e^x$

   (g) $f(x) = \dfrac{e^x - e^{-x}}{2}$

   (h) $f(x) = 5 - \ln(x - 2)^2,\ x \neq 2$

4. Sketch the graphs of the following functions.

   (a) $f(x) = |x^2 - 2x - 3|$

   (b) $f(x) = x^2 - 4|x| + 3$

   (c) $f(x) = ||x - 2| - 4|$

   (d) $f(x) = |3 - |x^2 - 1||$

5. Sketch the graphs of the following piece-wise defined functions.

   (a) $f(x) = \begin{cases} 2x + 5, & x < -1 \\ x^2 - 1, & x \geq -1 \end{cases}$

   (b) $f(x) = \begin{cases} x, & x < -2 \\ -x, & -2 \leq x < 2 \\ x, & x \geq 2 \end{cases}$

(c) $f(x) = \begin{cases} 1 - x^2, & x < 1 \\ \ln x, & x \geq 1 \end{cases}$

(d) $f(x) = \begin{cases} 1 - |x+3|, & x < -2 \\ 2 - |x|, & -2 \leq x < 2 \\ 1 - |x-3|, & x \geq 2 \end{cases}$

Chapter 2: Derivatives

1. Evaluate the following limits

(a) $\lim_{x\to 2}(x^3-5x+4)$

(b) $\lim_{x\to 3}\dfrac{x^2-2x-3}{x-3}$

(c) $\lim_{x\to -1}\dfrac{x^2+5}{x+2}$

(d) $\lim_{x\to 2}\dfrac{x^2+3x-10}{x^2-x-2}$

(e) $\lim_{x\to 4}\dfrac{2-\sqrt{x}}{4-x}$

(f) $\lim_{x\to 0}\dfrac{x}{\sqrt{x+4}-2}$

(g) $\lim_{x\to -2}\dfrac{x^3+8}{x^4-16}$

(h) $\lim_{h\to 0}\dfrac{\sqrt{h+16}-4}{h}$

(i) $\lim_{h\to -3}\dfrac{\frac{1}{h}+\frac{1}{3}}{h+3}$

(j) $\lim_{w\to 0}\dfrac{\sqrt{4+w}-\sqrt{4-w}}{w}$

(k) $\lim_{t\to 0}\left(\dfrac{1}{2t}-\dfrac{1}{t^2+2t}\right)$

(l) $\lim_{z\to 0}\left(\dfrac{1}{z\sqrt{1+z}}-\dfrac{1}{z}\right)$

(m) $\lim_{x\to 0}\dfrac{\sin^2 x}{x^2}$

(n) $\lim_{x\to 0}\dfrac{\sin 3x}{4x}$

(o) $\lim_{x\to 0}\dfrac{\sin^2 x}{1-\cos x}$

(p) $\lim_{t\to 0}\dfrac{\ln(1+t)}{t}$

(q) $\lim_{h\to 0}\dfrac{e^h-1}{h}$

2. Evaluate the following limits.

(a) $\lim_{x\to\infty}\dfrac{2x+5}{x^2+3}$

(b) $\lim_{x\to\infty}\dfrac{3x+1}{5x-4}$

(c) $\lim_{x\to\infty}\dfrac{6x^2+2x-5}{2x^2-4x+1}$

(d) $\lim_{x\to+\infty}\dfrac{\sqrt{9x^4-3x+2}}{x^2-2x+5}$

(e) $\lim_{x\to+\infty}\left(\sqrt{4x^2+5x}-2x\right)$

(f) $\lim_{x\to-\infty}\left(x+\sqrt{x^2+4x}\right)$

(g) $\lim_{x\to+\infty}\dfrac{e^x+x^2}{e^x-x^2}$

(h) $\lim_{x\to\infty}\dfrac{\sin x}{x}$

(i) $\lim_{x\to+\infty}\dfrac{\cos x}{(\ln x)^2}$

3. Use definition to evaluate the derivatives of the following functions.

(a) $y=3x-2$

(b) $y=(x+1)^2$

(c) $y=x^4$

(d) $y=3\sqrt{x}$

(e) $y=\sqrt{x+2}$

(f) $y=\dfrac{1}{x^2}$

(g) $y=\dfrac{1}{\sqrt{x}}$

(h) $y=\cos x$

(i) $y=\ln x$

(j) $y=e^x$

4. For each of the following functions, determine whether it is differentiable at $x=0$. Find $f'(0)$ if it is.

(a) $f(x)=x^{\frac{4}{3}}$

(b) $f(x)=|\sin x|$

(c) $f(x)=x|x|$

(d) $f(x)=\begin{cases}5-2x, & \text{when } x<0\\ x^2-2x+5, & \text{when } x\ge 0\end{cases}$

(e) $f(x)=\begin{cases}1+3x-x^2, & \text{when } x<0\\ x^2+3x+2, & \text{when } x\ge 0\end{cases}$

(f) $f(x)=\begin{cases}e^{-\frac{1}{x^2}}, & \text{when } x\ne 0\\ 0, & \text{when } x=0\end{cases}$

5. Find the first derivatives of the following functions.

(a) $y = x^3 - 4x + 3$

(b) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

(c) $y = x^2 e^{5x}$

(d) $y = \cos x \ln x$

(e) $y = \sin x \cos x$

(f) $y = 3\sec x - \tan x$

(g) $y = x \cot x$

(h) $y = \dfrac{3x-4}{x+2}$

(i) $y = \dfrac{x^2+1}{x+1}$

(j) $y = \dfrac{\sin x}{x}$

(k) $y = \dfrac{\tan x}{\sqrt{x}}$

(l) $y = (x^2+1)^7$

(m) $y = \sqrt{x^4+1}$

(n) $y = \cos(x^2)$

(o) $y = xe^{x^3+x}$

(p) $y = \ln(\ln x)$

(q) $y = e^{\sin x}$

(r) $y = \dfrac{x}{\sqrt{1+x^2}}$

(s) $y = \ln(x + \sqrt{1+x^2})$

(t) $y = \sqrt{x + \sqrt{x}}$

(u) $y = \sin^{-1}\sqrt{x}$

(v) $y = \cos\tan^{-1} x$

6. Find the first derivatives of the following functions.

(a) $y = 3^x$

(b) $y = 2^{\cos x}$

(c) $y = x^x$

(d) $y = x^{\sqrt{x}}$

(e) $y = (\cos x)^{\sin x}$

(f) $y = x^{x^x}$

7. Find $\dfrac{dy}{dx}$ for the following implicit functions.

(a) $x^2 + y^2 = 4$

(b) $x^3 y + xy^2 = 1$

(c) $x^3 + y^3 = 2xy$

(d) $xe^{xy} = 1$

(e) $\sin(xy) = (x+y)^2$

(f) $\cos\left(\dfrac{y}{x}\right) = \ln(x+y)$

8. Find $\dfrac{d^2y}{dx^2}$ for the following functions.

(a) $y = \sqrt{x}e^{x^2}$

(b) $y = \dfrac{x}{\sqrt{1+x^2}}$

(c) $y = (\ln x)^2$

(d) $y = \sec x$

(e) $y = \tan^{-1} x$

(f) $x^2 + y^3 = 1$

(g) $x^4 y - 3x^2 y^3 = 5$

(h) $y = 3^{x^2}$

(i) $y = x^{\ln x}$

9. Prove that the Chebyshev polynomials

$$T_m(x) = \frac{1}{2^{m-1}}\cos(m\cos^{-1} x),\ \ m = 0, 1, 2, \cdots$$

satisfy

$$(1-x^2)T_m''(x) - xT_m'(x) + m^2 T_m(x) = 0$$

10. Prove that the Legendre polynomials

$$P_m(x) = \frac{1}{2^m m!}\frac{d^m}{dx^m}(x^2-1)^m \ \ m = 0, 1, 2, \cdots$$

satisfy

$$(1-x^2)P_m''(x) - 2xP_m'(x) + m(m+1)P_m(x) = 0$$

11. Show that the function

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is not differentiable at $x = 0$.

12. Show that the function

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is differentiable at $x = 0$ but $f'(x)$ is not continuous at $x = 0$.

13. Find the absolute maximum and absolute minimum of the following functions on the given intervals.

(a) $f(x) = x^3 - 6x^2 + 9x - 6$; $[2, 5]$

(b) $f(x) = x^4 - 4x^3 + 5$; $[0, 4]$

(c) $f(x) = x + \dfrac{16}{x}$; $(0, +\infty)$

(d) $f(x) = \dfrac{x^2}{x^2 + 1}$; $(-\infty, +\infty)$

(e) $f(x) = 2x^2 - \ln x$; $(0, 3]$

(f) $f(x) = \sqrt{2 + x - x^2}$; $(-\infty, +\infty)$

(g) $f(x) = x^2 e^{-x}$; $[-1, +\infty)$

(h) $f(x) = x^{\frac{1}{x}}$; $(0, +\infty)$

(i) $f(x) = 3 - (x - 2)^{\frac{2}{3}}$; $[0, 10]$

(j) $f(x) = x^{\frac{2}{3}}(x - 1)$; $[-1, 1]$

14. Sketch the graph of each of the following functions. Show clearly, if there is any, the following:

- x-intercepts and y-intercepts
- vertical and horizontal asymptotes
- intervals of increase or decrease
- local extremum points
- intervals of concavity
- inflection points

(a) $y = \dfrac{4x + 15}{2x - 5}$

(b) $y = 27 + 6x^2 - x^4$

(c) $y = x^4 + 4x^3 + 12$

(d) $y = \dfrac{1}{x^2 - 16}$

(e) $y = \dfrac{x}{x^2 + 9}$

(f) $y = \dfrac{x + 3}{(x - 1)^2}$

(g) $y = \dfrac{x^2}{x^2 + 3}$

(h) $y = \dfrac{(x - 2)^2}{x^2 + 4}$

(i) $y = \dfrac{x^2 - 2x - 4}{x^2}$

Chapter 3: Taylor's theorem

1. Let $f(x) = 1 - \sqrt[3]{x^2}$. We have $f(-1) = f(1) = 0$ and $f'(x) \neq 0$ for any x. Explain why this does not contradict the mean value theorem.

2. Let $f : [a, b] \to \mathbb{R}$ be a function which is continuous on $[a, b]$ and differentiable on (a, b). Suppose $f'(x) = 0$ for any $x \in (a, b)$. Prove that f is a constant function.

3. Let $f : [a, b] \to \mathbb{R}$ be a function which is continuous on $[a, b]$ and differentiable on (a, b). Suppose $f'(x) \geq 0$ for any $x \in (a, b)$ and $f'(x) = 0$ at finitely many points. Prove that f is strictly increasing on $[a, b]$. (A function f is strictly increasing if $f(x) < f(y)$ for any $x < y$.)

4. Let $f : [a, b] \to \mathbb{R}$ be a function which is continuous on $[a, b]$ and the n-th derivative of f exists on (a, b). Suppose there exists $a = a_0 < a_1 < a_2 < \cdots < a_n = b$ such that $f(a_k) = 0$ for $k = 0, 1, 2, \cdots n$. Prove that there exists $c \in (a, b)$ such that $f^{(n)}(c) = 0$.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function and suppose

$$\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x)$$

Prove that there exists $c \in \mathbb{R}$ such that $f'(c) = 0$.

6. Let $a < c < b$. Let $f : (a, b) \to \mathbb{R}$ is a continuous function. Suppose f is differentiable on $(a, b) \setminus \{c\}$ and $\lim\limits_{x\to c} f'(x)$ exists. Prove that f is differentiable at c and $f'(c) = \lim\limits_{x\to c} f'(x)$.

7. Let f be a function which is differentiable at x for any $x \in [a, b]$.
   (a) Prove that if $f'(a) < 0$ and $f'(b) > 0$, then there exits $c \in (a, b)$ such that $f'(c) = 0$. (Hint: By extreme value theorem, f attains its minimum on $[a, b]$.)
   (b) Prove that if $f'(a) < f'(b)$, then for any $f'(a) < K < f'(b)$, there exists $c \in (a, b)$ such that $f'(c) = K$. (This may be considered as the intermediate value theorem for derivatives of functions. Note that f' may not be continuous.)

8. Prove the following case of L'Hôpital rule: Let $f(x)$ and $g(x)$ be differentiable functions such that $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = 0$ and $\lim_{x\to\infty} \frac{f'(x)}{g'(x)} = L$. Then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$.

9. Use L'Hospital rule to evaluate the following limits.

(a) $\displaystyle\lim_{x\to0} \frac{\sin 3x}{\sin 5x}$ (d) $\displaystyle\lim_{x\to0} \frac{1 - x\cot x}{x\sin x}$ (g) $\displaystyle\lim_{x\to0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

(b) $\displaystyle\lim_{x\to0} \frac{\sin^2 x}{1 - \cos x}$ (e) $\displaystyle\lim_{x\to0} \frac{\sinh x - \sin x}{x(\cosh x - \cos x)}$ (h) $\displaystyle\lim_{x\to0} \frac{e^x - x - 1}{1 - \cosh x}$

(c) $\displaystyle\lim_{x\to0} \frac{x - \sin^3 x}{2\sin x - \sin 2x}$ (f) $\displaystyle\lim_{x\to0} \frac{\ln\cos 2x}{\ln\cos x}$ (i) $\displaystyle\lim_{x\to0} \frac{e - (1 + x)^{\frac{1}{x}}}{x}$

(j) $\lim_{x\to 0} \frac{2^x - 1}{x}$ (k) $\lim_{x\to 0^+} x^{\frac{1}{1+\ln x}}$ (l) $\lim_{x\to 1} x^{\frac{1}{1-x}}$

10. Find the Taylor polynomials centered at 0 of the following functions up to the term in x^3.

    (a) $\frac{1}{(1+x)^2}$
    (b) $\sqrt{1-x}$
    (c) $(1+\sin x)^2$
    (d) $\ln\cos x$
    (e) $\frac{1}{\cosh x}$
    (f) $\sin^{-1} x$

11. Find the Taylor polynomials of degree 3 of the following functions at the given center c.

    (a) $\frac{1}{\sqrt{x}}$; $c = 1$
    (b) $\ln x$; $c = e$
    (c) $\tan x$; $c = \frac{\pi}{4}$

12. For each of the following functions $f(x)$ and value a, us the Taylor polynomial of degree 3 to approximate the value of $f(a)$ and state the maximum possible error.

    (a) $f(x) = \tan^{-1} x$; $a = 1$
    (b) $f(x) = \cos x$; $a = 0.5$
    (c) $f(x) = \ln(1+x)$; $a = 1$
    (d) $f(x) = \sqrt{4+x}$; $a = 0.1$

13. Suppose the Taylor series of $f(x)$ is $1 + a_1x + a_2x^2 + a_3x^3 + \cdots$. Find the Taylor polynomial of $\frac{1}{f}$ up to the term in x^3 in terms of a_1, a_2, a_3 by

    (a) expanding the product of the series for f and $\frac{1}{f}$.
    (b) finding the first three derivatives of $\frac{1}{f}$.

14. Suppose the degree 3 Taylor polynomials of $f(x)$ and $g(x)$ are $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $q(x) = b_1x + b_2x^2 + b_3x^3$ respectively. Find the Taylor polynomial of $f \circ g$ up to the term in x^3 in terms of $a_0, a_1, a_2, a_3, b_1, b_2, b_3$ by

    (a) expanding the polynomial $p(q(x))$.
    (b) finding the first three derivatives of $f \circ g$.

15. Prove that the Taylor series of the function

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is identically 0.

Chapter 4: Integration

1. Evaluate the following indefinite integrals.

   (a) $\int (3-x^2)^3 dx$ (c) $\int \frac{x+1}{\sqrt{x}}\,dx$ (e) $\int 3\csc^2 x dx$

   (b) $\int x^2(5-x)^4 dx$ (d) $\int \left(8t - \frac{2}{t^{\frac{1}{4}}}\right) dt$ (f) $\int 4\tan\theta\sec\theta d\theta$

2. Use a suitable substitution to evaluate the following integral.

   (a) $\int \frac{dx}{\sqrt{2-5x}}$ (e) $\int \frac{xdx}{(1+x^2)^2}$ (i) $\int \frac{e^x dx}{2+e^x}$

   (b) $\int \frac{e^{3x}+1}{e^x+1}\,dx$ (f) $\int \frac{dx}{\sqrt{x}(1+x)}$ (j) $\int \frac{dx}{e^x+e^{-x}}$

   (c) $\int \frac{x}{\sqrt{1-x^2}}\,dx$ (g) $\int \frac{1}{x^2}\sin\frac{1}{x}\,dx$ (k) $\int \tan x dx$

   (d) $\int x^2\sqrt[3]{1+x^3}\,dx$ (h) $\int xe^{-x^2}dx$ (l) $\int \frac{dx}{1+e^x}$

3. Evaluate the following definite integrals.

   (a) $\int_1^3 \frac{2x^3-5}{x^2}dx$ (c) $\int_0^1 \frac{5x}{(4+x^2)^2}dx$ (e) $\int_0^{\frac{\pi}{4}} \sec^2 x\tan x dx$

   (b) $\int_0^1 x\sqrt{1-x^2}dx$ (d) $\int_0^{\pi} \cos^2 x\sin x dx$ (f) $\int_1^4 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}dx$

4. Find the area of the regions bounded by the graphs of the given functions.

   (a) $y = 4 - x^2$; x-axis (d) $y = x^2$; $y = x - 2$; x-axis

   (b) $y = 3 - x^2$; $y = -x - 3$ (e) $y = \sqrt{x}$; $y = x - 2$; x-axis

   (c) $y = x^2 - 4$; $y = -x^2 - 2x$ (f) $x + y^2 = 4$; $x + y = 2$

Chapter 5: Further techniques of integration

1. Evaluate

(a) $\int \frac{dx}{1-\cos x}$
(b) $\int \sin^5 x \cos x dx$
(c) $\int \sin 3x \sin 5x dx$
(d) $\int \cos\frac{x}{2}\cos\frac{x}{3}\,dx$
(e) $\int \cos^3 x dx$
(f) $\int \sin^4 x dx$
(g) $\int \frac{dx}{\cos x \sin^2 x}$
(h) $\int \frac{\sin x \cos^3 x}{1+\cos^2 x}, dx$
(i) $\int \tan^5 x dx$
(j) $\int \frac{dx}{\sin^4 x \cos^4 x}, dx$
(k) $\int \sin 5x \cos x dx$
(l) $\int \cos x \cos 2x \cos 3x dx$

2. Evaluate

(a) $\int \ln x dx$
(b) $\int x^2 \ln x dx$
(c) $\int \left(\frac{\ln x}{x}\right)^2 dx$
(d) $\int xe^{-x}dx$
(e) $\int x^2 e^{-2x}dx$
(f) $\int x \cos x dx$
(g) $\int x^2 \sin 2x dx$
(h) $\int \sin^{-1} x dx$
(i) $\int x \tan^{-1} x dx$
(j) $\int \ln(x+\sqrt{1+x^2})dx$
(k) $\int x \sin^2 x dx$
(l) $\int \sin(\ln x)dx$

3. Prove the following reduction formulas.

(a) $I_n = \int x^n e^{ax}dx$; $I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a}I_{n-1}$, $n \geq 1$
(b) $I_n = \int \sin^n x dx$; $I_n = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n}I_{n-2}$, $n \geq 2$
(c) $I_n = \int \cos^n x dx$; $I_n = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}I_{n-2}$, $n \geq 2$
(d) $I_n = \int \frac{1}{\sin^n x}\,dx$; $I_n = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1}I_{n-2}$, $n \geq 2$
(e) $I_n = \int x^n \cos x dx$; $I_n = x^n \sin x + nx^{n-1}\cos x - n(n-1)I_{n-2}$, $n \geq 2$
(f) $I_n = \int \frac{dx}{(x^2-a^2)^n}$; $I_n = -\frac{x}{2a^2(n-1)(x^2-a^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)}I_{n-1}$, $n \geq 1$
(g) $I_n = \int \frac{x^n dx}{\sqrt{x+a}}$; $I_n = \frac{2x^n\sqrt{x+a}}{2n+1} - \frac{2an}{2n+1}I_{n-1}$, $n \geq 1$

4. Use trigonometric substitution to evaluate the following integrals.

(a) $\displaystyle\int \frac{x^2}{1+x^2}\,dx$ (c) $\displaystyle\int \sqrt{\frac{1+x}{1-x}}\,dx$ (e) $\displaystyle\int \frac{x^2dx}{\sqrt{9-x^2}}$

(b) $\displaystyle\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$ (d) $\displaystyle\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$ (f) $\displaystyle\int \frac{dx}{\sqrt{4+x^2}}$

5. Evaluate the following integrals of rational functions.

(a) $\displaystyle\int \frac{x^2dx}{1-x^2}$ (e) $\displaystyle\int \frac{dx}{(x^2-2)(x^2+3)}$ (i) $\displaystyle\int \frac{dx}{(x+1)(x^2+1)}$

(b) $\displaystyle\int \frac{x^3}{3+x}\,dx$ (f) $\displaystyle\int \frac{x^2+1}{(x+1)^2(x-1)},dx$ (j) $\displaystyle\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$

(c) $\displaystyle\int \frac{(1+x)^2}{1+x^2}\,dx$ (g) $\displaystyle\int \frac{x^2}{(x^2-3x+2)^2},dx$ (k) $\displaystyle\int \frac{4-2x}{(x^2+1)(x-1)^2}\,dx$

(d) $\displaystyle\int \frac{dx}{x^2+2x-3}$ (h) $\displaystyle\int \frac{x^2+5x+4}{x^4+5x^2+4},dx$ (l) $\displaystyle\int \frac{dx}{x(x^2+1)^2}$

6. Use t-substitution to evaluate the following integrals.

(a) $\displaystyle\int \frac{dx}{\sin^3 x}$ (b) $\displaystyle\int \frac{dx}{1+\sin x}$ (c) $\displaystyle\int \frac{dx}{\sin x\cos^4 x}$

7. Evaluate the following improper integrals.

(a) $\displaystyle\int_4^{\infty} \frac{dx}{x^2}$ (d) $\displaystyle\int_2^{\infty} \frac{dx}{x^2+x-2}$ (g) $\displaystyle\int_0^{\infty} \frac{\tan^{-1} x}{(1+x^2)^{\frac{3}{2}}}\,dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ (e) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+x+1)^2}$ (h) $\displaystyle\int_0^{\infty} e^{-x}\cos x dx$

(c) $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}}$ (f) $\displaystyle\int_0^{\infty} \frac{dx}{1+x^3}$ (i) $\displaystyle\int_0^{\frac{\pi}{2}} \ln(\sin x)dx$

8. Determine whether the following improper integrals are convergent.

(a) $\displaystyle\int_0^{\infty} \frac{x^2dx}{x^4-x^2+1}$ (c) $\displaystyle\int_0^{1} \frac{dx}{\ln x}$ (e) $\displaystyle\int_0^{\infty} \frac{\ln(1+x)}{\sqrt{x}}\,dx$

(b) $\displaystyle\int_1^{\infty} \frac{dx}{x\sqrt[3]{x^2+1}}$ (d) $\displaystyle\int_2^{\infty} \frac{dx}{x\ln x}$ (f) $\displaystyle\int_0^{\frac{\pi}{2}} \tan x dx$

9. Evaluate

(a) $\displaystyle\int f(x)dx$ where $f(x)=\begin{cases}1, & \text{if } x<0\\ 2x+1, & \text{if } x\geq 0\end{cases}$

(b) $\displaystyle\int f(x)dx$ where $f(x)=\begin{cases}4x-1, & \text{if } x<3\\ x^2+1, & \text{if } x\geq 3\end{cases}$

(c) $\int |x| dx$

(d) $\int |x^2 - 1| dx$

(e) $\int |x^2 - x| dx$

(f) $\int x^2 \sqrt[3]{1-x}\, dx$

(g) $\int x^5 (2 - 5x^3)^{\frac{2}{3}} dx$

(h) $\int e^x \cos x dx$

(i) $\int \frac{x dx}{\cos^2 x}$

(j) $\int \frac{dx}{\sqrt{25x^2 - 4}}$

(k) $\int \frac{dx}{(x^2+1)^3}$

(l) $\int \frac{x^2}{(x^2+2x+2)^2}\, dx$

(m) $\int \frac{4dx}{(4x^2+1)^2}$

(n) $\int \sqrt{\frac{4-x}{x}}\, dx$

(o) $\int \frac{8dx}{x^2\sqrt{4-x^2}}\, dx$

(p) $\int \frac{dx}{(\cos x + \sin x)^2}$

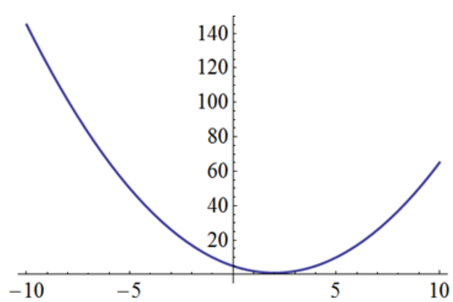
(q) $\int \sqrt{1 + \sin x}\, dx$

Answers
Chapter 1: Functions

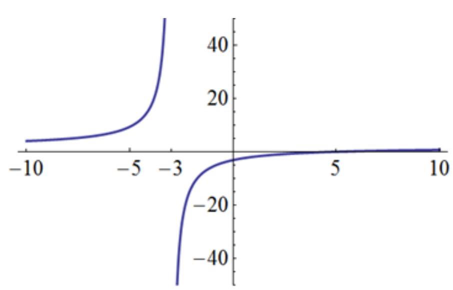
1. (a) maximum domain $= \mathbb{R}$, range $= \mathbb{R}$
   (b) maximum domain $= (-\infty, \frac{7}{2}]$, range $= [0, \infty)$
   (c) maximum domain $= (-\infty, -2) \cup (5, \infty)$, range $= (-1, \infty)$
   (d) maximum domain $= \mathbb{R}$, range $= [-\frac{\sqrt{5}}{10-2\sqrt{5}}, \frac{\sqrt{5}}{10+2\sqrt{5}}]$
   (e) maximum domain $= (-\infty, -2) \cup (2, \infty)$, range $= (0, \infty)$
   (f) maximum domain $= \mathbb{R} \setminus \{n\pi, n \in \mathbb{Z}\}$, range $= \mathbb{R} \setminus \{0\}$
   (g) maximum domain $= \mathbb{R} \setminus \{(n + \frac{3}{4})\pi, n \in \mathbb{Z}\}$, range $= \mathbb{R} \setminus \{0\}$
   (h) maximum domain $= (3, \infty)$, range $= \mathbb{R}$
   (i) maximum domain $= \mathbb{R} \setminus \{e\}$, range $= \mathbb{R} \setminus \{0\}$
   (j) maximum domain $= [e^{-3}, \infty)$, range $= [0, \infty)$
   (k) maximum domain $= (1, \infty)$, range $= \mathbb{R}$
   (l) maximum domain $= [1 - e^2, 1 - e^{-2}]$, range $= [0, \infty)$

2. (a) Injective, Surjective, Bijective
   (b) Not injective, Not surjective, Not bijective
   (c) Injective, Surjective, Bijective
   (d) Not injective, Not surjective, Not bijective
   (e) Injective, Not surjective, Not bijective
   (f) Injective, Not surjective, Not bijective
   (g) Injective, Surjective, Bijective
   (h) Injective, Not surjective, Not bijective

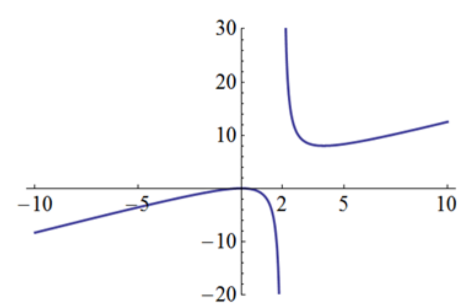
3. (a)



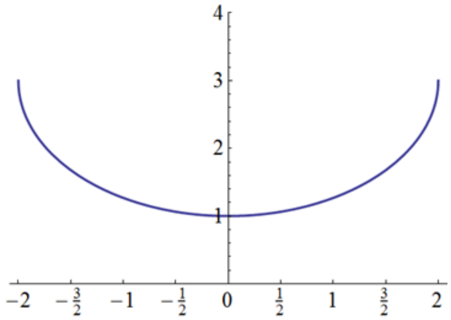
(b)



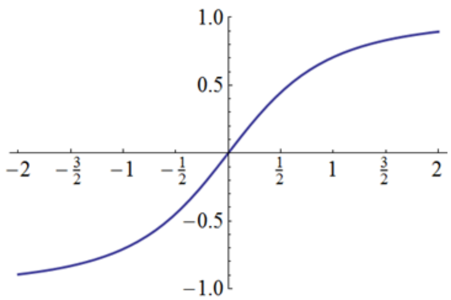
(c)



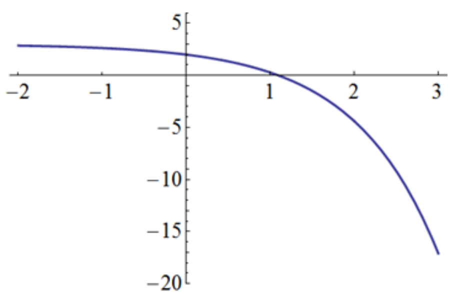
(d)



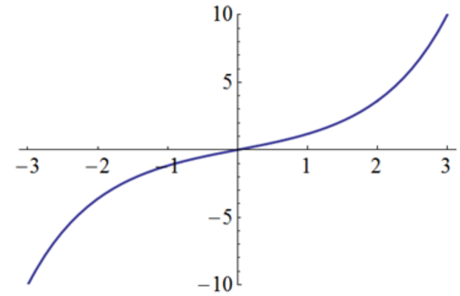
(e)



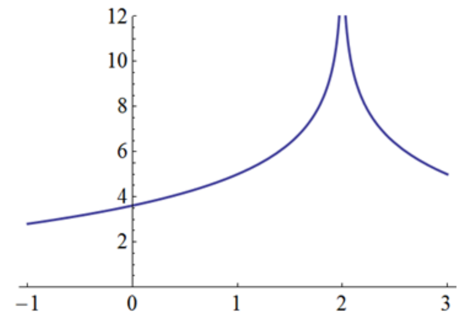
(f)



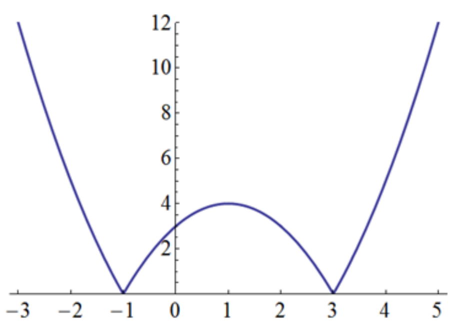
(g)



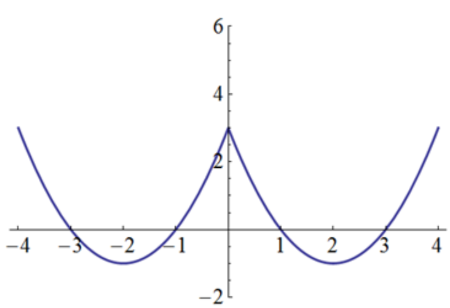
(h)



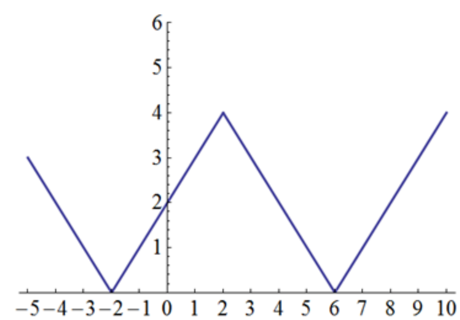
4. (a)



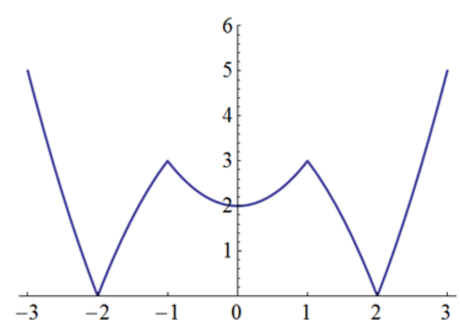
(b)



(c)

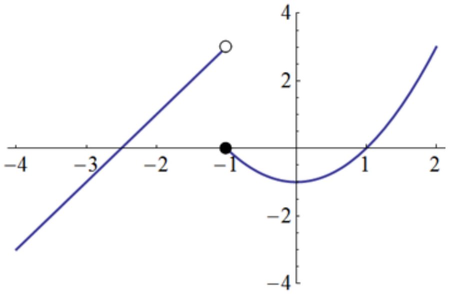


(d)

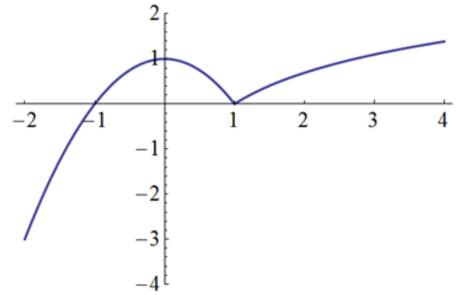


5.

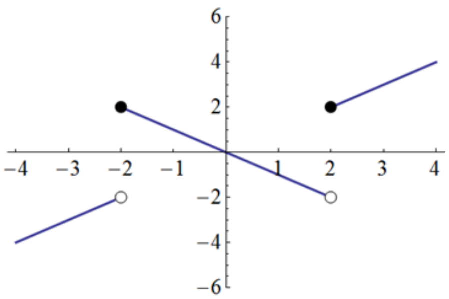
(a)



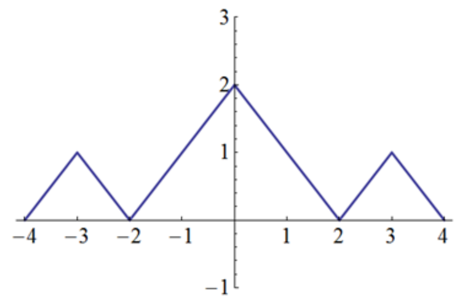
(c)



(b)



(d)



Chapter 2: Derivatives

1. (a) 2
   (b) 4
   (c) 6
   (d) $\frac{7}{3}$
   (e) $\frac{1}{4}$
   (f) 4
   (g) $-\frac{3}{8}$
   (h) $\frac{1}{8}$
   (i) $-\frac{1}{9}$
   (j) $\frac{1}{2}$
   (k) $\frac{1}{4}$
   (l) $-\frac{1}{2}$
   (m) 1
   (n) $\frac{3}{4}$
   (o) 2
   (p) 1
   (q) 1

2. (a) 0
   (b) $\frac{3}{5}$
   (c) 3
   (d) 3
   (e) $\frac{5}{4}$
   (f) -2
   (g) 1
   (h) 0
   (i) 0

3. (a) $y' = 3$
   (b) $y' = 2(x+1)$
   (c) $y' = 4x^3$
   (d) $y' = \frac{3}{2\sqrt{x}}$
   (e) $y' = \frac{1}{2\sqrt{x+2}}$
   (f) $y' = -\frac{2}{x^3}$
   (g) $y' = -\frac{1}{x^{\frac{3}{2}}}$
   (h) $y' = -\sin x$
   (i) $y' = \frac{1}{x}$
   (j) $y' = e^x$

4. (a) 0
   (b) not differentiable
   (c) 0
   (d) -2
   (e) not differentiable
   (f) 0

5. (a) $3x^2 - 4$
   (b) $(x-1)/2x^{3/2}$
   (c) $(5x^2 + 2x)e^{5x}$
   (d) $-\sin x \, \ln x + \cos x/x$
   (e) $\cos 2x$
   (f) $(3\sin x - 1)/\cos^2 x$
   (g) $\cot x - x\csc^2 x$
   (h) $10/(x+2)^2$
   (i) $1 - 2/(x+1)^2$
   (j) $(x\cos(x) - \sin x)/x^2$
   (k) $(2x\tan^2 x - \tan x + 2x)/2x^{3/2}$
   (l) $14x(x^2+1)^6$
   (m) $2x^3/\sqrt{x^4+1}$
   (n) $-2x\sin(x^2)$
   (o) $(3x^3 + x^2 + 1)e^{x^3+x}$
   (p) $1/(x\ln x)$
   (q) $\cos(x)e^{\sin x}$
   (r) $1/(x^2+1)^{3/2}$
   (s) $1/\sqrt{x^2+1}$
   (t) $(1+2\sqrt{x})/(4\sqrt{x}\sqrt{x+\sqrt{x}})$
   (u) $1/\left(2\sqrt{x(1-x)}\right)$
   (v) $-x/(x^2+1)^{3/2}$

6. (a) $3^x \ln 3$
   (b) $-\ln 2 \sin(x) 2^{\cos(x)}$
   (c) $(\ln(x) + 1)x^x$
   (d) $x^{\sqrt{x}-1/2}(\ln(x)/2 + 1)$
   (e) $(\cos x)^{\sin x - 1}(\cos^2 x + \ln(\cos x)\cos^2 x - 1)$
   (f) $x^x x^{x^x}(x(\ln x)^2 + x\ln x + 1)/x$

7. (a) $-\dfrac{x}{y}$
   (b) $-\dfrac{y^2 + 3x^2 y}{x^3 + 2xy}$
   (c) $\dfrac{3x^2 - 2y}{2x - 3y^2}$
   (d) $-\dfrac{1}{x^2}$

(e) $\dfrac{2x+2y}{x\cos(xy)-2y-2x}$

(f) $\dfrac{\frac{y\sin(y/x)}{x^2}-\frac{1}{x+y}}{\frac{1}{x+y}+\frac{\sin(y/x)}{x}}$

8. (a) $x^{-\frac{3}{2}}e^{x^2}+x^{\frac{1}{2}}e^{x^2}+4x^{\frac{5}{2}}e^{x^2}$
   (b) $-3x(1+x^2)^{-\frac{5}{2}}$
   (c) $-\dfrac{2}{x^2}\ln x+\dfrac{2}{x^2}$
   (d) $\dfrac{\cos^3 x+2\sin^2 x\cos x}{\cos^4 x}$
   (e) $\dfrac{-2x}{(1+x^2)^2}$
   (f) $-\dfrac{8x^2+6y^3}{9y^5}$
   (g) $\dfrac{10(2x^6y+3x^4y^3-81y^7)}{x^2(x^2-9y^2)^3}$
   (h) $2\cdot 3^{x^2}\ln 3+4\cdot x^2 3^{x^2}(\ln 3)^2$
   (i) $\dfrac{4y}{x^2}(\ln x)^2+\dfrac{2y}{x^2}(1-\ln x)$

9. Prove that the Chebyshev polynomials

$$T_m(x)=\frac{1}{2^{m-1}}\cos(m\cos^{-1}x),\ \ m=0,1,2,\cdots$$

satisfy

$$(1-x^2)T_m''(x)-xT_m'(x)+m^2T_m(x)=0$$

Proof. By direct computations,

$$T_m'(x)=\frac{m\sin(m\cos^{-1}x)}{2^{m-1}\sqrt{1-x^2}}$$

and

$$T_m''(x)=\frac{m}{2^{m-1}}\left(\frac{x\sin(m\cos^{-1}x)}{(1-x^2)^{\frac{3}{2}}}-\frac{m}{1-x^2}\cos(m\cos^{-1}x)\right).$$

Hence,

$$\begin{aligned}&(1-x^2)T_m''(x)-xT_m'(x)+m^2T_m(x)\\=&(1-x^2)\frac{m}{2^{m-1}}\left(\frac{x\sin(m\cos^{-1}x)}{(1-x^2)^{\frac{3}{2}}}-\frac{m}{1-x^2}\cos(m\cos^{-1}x)\right)\\&-x\frac{m\sin(m\cos^{-1}x)}{2^{m-1}\sqrt{1-x^2}}+m^2\frac{1}{2^{m-1}}\cos(m\cos^{-1}x)\\=&0.\end{aligned}$$

□

10. Prove that the Legendre polynomials

$$P_m(x)=\frac{1}{2^m m!}\frac{d^m}{dx^m}(x^2-1)^m\ \ m=0,1,2,\cdots$$

satisfy

$$(1-x^2)P_m''(x)-2xP_m'(x)+m(m+1)P_m(x)=0$$

Proof. Let $g(x) = (x^2-1)^m$, then $P_m(x) = \frac{1}{2^m m!}\frac{d^m}{dx^m}g(x)$.

Because
$$-\frac{d}{dx}(x^2-1)^m = 2mx(x^2-1)^{m-1},$$
therfore,
$$-\frac{d(x^2-1)^m}{dx}(x^2-1) + 2mx(x^2-1)^m = 0.$$
We get
$$\begin{aligned} & -g'(x)(x^2-1) + 2mxg(x) = 0 \\ \Rightarrow \quad & \frac{d^{m+1}}{dx^{m+1}}\big(-g'(x)(x^2-1) + 2mxg(x)\big) = 0. \end{aligned}$$

Apply Leibniz's rule,
$$\begin{aligned} \Rightarrow \quad & -\big(g^{(m+2)}(x)(x^2-1) + C_1^{m+1}g^{(m+1)}(x)\cdot 2x + C_2^{m+1}g^{(m)}(x)\cdot 2\big) \\ & \qquad + 2m\big(g^{(m+1)}(x)\cdot x + C_1^{m+1}g^{(m)}(x)\big) = 0 \\ \Rightarrow \quad & (1-x^2)g^{(m+2)}(x) - 2xg^{(m+1)}(x) + m(m+1)g^{(m)}(x) = 0 \\ \Rightarrow \quad & \frac{1}{2^m m!}\big((1-x^2)g^{(m+2)}(x) - 2xg^{(m+1)}(x) + m(m+1)g^{(m)}(x)\big) = 0 \\ \Rightarrow \quad & (1-x^2)P_m''(x) - 2xP_m'(x) + m(m+1)P_m(x) = 0. \end{aligned}$$

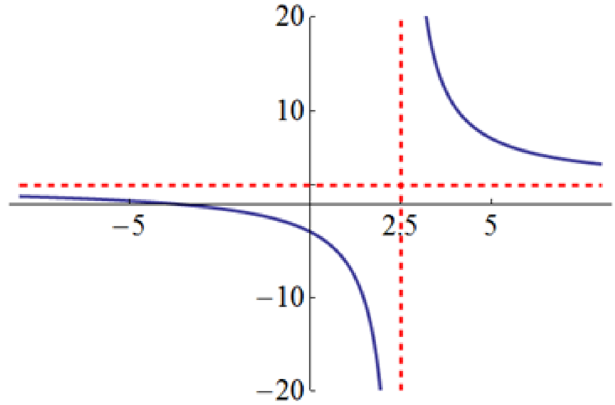
□

11.

12.

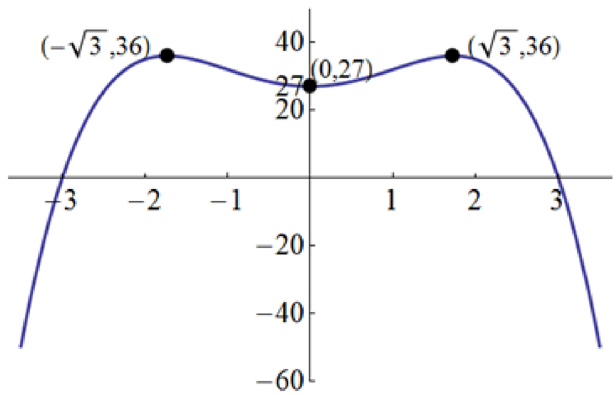
13. (a) Maximum: $f(5) = 14$; Minimum: $f(3) = -6$
    (b) Maximum: $f(0) = f(4) = 5$; Minimum: $f(3) = -22$
    (c) No absolute maximum; Minimum: $f(4) = 8$
    (d) Maximum: $f(0) = 0$; No absolute minimum
    (e) No absolute maximum; Minimum: $f(\frac{1}{2}) = \frac{1}{2} + \ln 2$
    (f) Maximum: $f(\frac{1}{2}) = \frac{3}{2}$; No absolute minimum
    (g) Maximum: $f(-1) = e$; Minimum: $f(0) = 0$
    (h) Maximum: $f(e) = e^{\frac{1}{e}}$; No absolute minimum
    (i) Maximum: $f(2) = 3$; Minimum: $f(10) = -1$
    (j) Maximum: $f(0) = f(1) = 0$; Minimum: $f(-1) = -2$

14. (a)



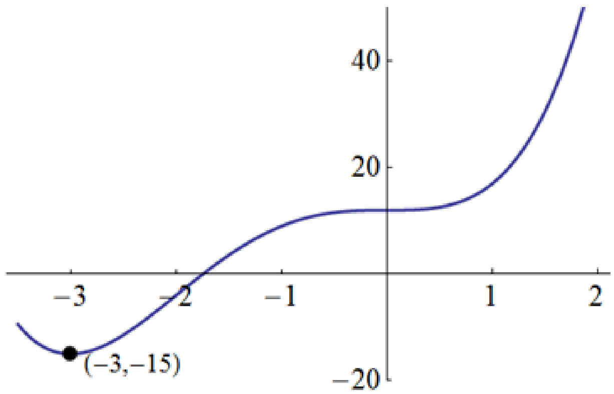
vertical asymptote(s): $x = 2.5$
horizontal asymptote(s): $y = 2$
interval(s) of increasing: none
interval(s) of decreasing: $(-\infty, 2.5), (2.5, \infty)$
local extremum points(s): none
interval(s) of concavity: $(-\infty, 2.5)$
inflection point(s): none

(b)



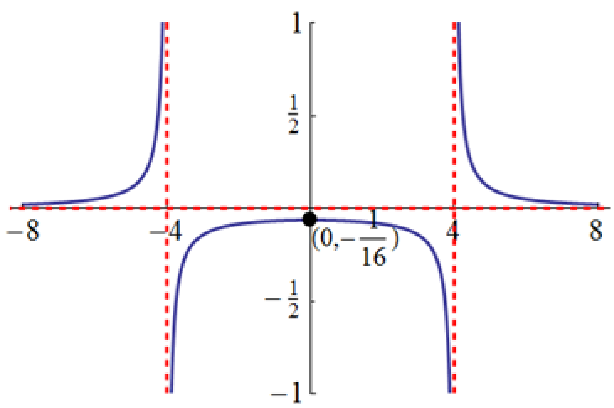
vertical asymptote(s): none
horizontal asymptote(s): none
interval(s) of increasing: $(-\infty, -\sqrt{3}], [0, \sqrt{3}]$
interval(s) of decreasing: $[-\sqrt{3}, 0], [\sqrt{3}, \infty)$
local extremum points(s): $x = -\sqrt{3}, 0, \sqrt{3}$
interval(s) of concavity: $(-\infty, -1], [1, \infty)$
inflection point(s): $x = -1, 1$

(c)



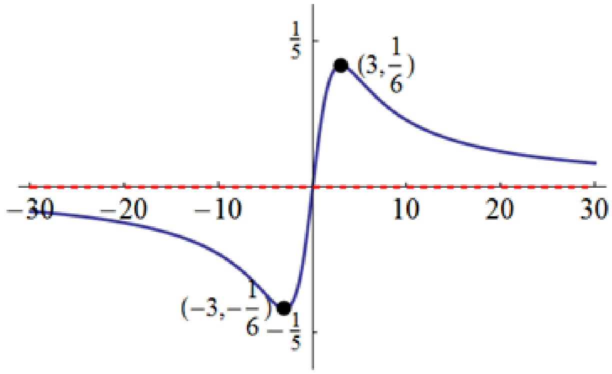
vertical asymptote(s): none
horizontal asymptote(s): none
interval(s) of increasing: $[-3, \infty)$
interval(s) of decreasing: $(-\infty, -3]$
local extremum points(s): $x = -3$
interval(s) of concavity: $[-2, 0]$
inflection point(s): $x = -2, 0$

(d)



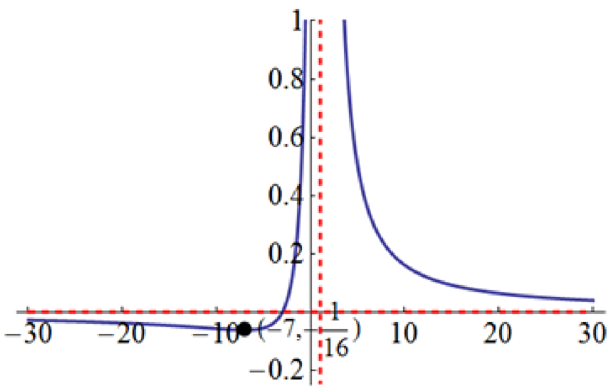
vertical asymptote(s): $x = -4, x = 4$
horizontal asymptote(s): $y = 0$
interval(s) of increasing: $(-\infty, -4), (-4, 0]$
interval(s) of decreasing: $[0, 4), (4, \infty)$
local extremum points(s): $x = 0$
interval(s) of concavity: $(-4, 4)$
inflection point(s): none

(e)



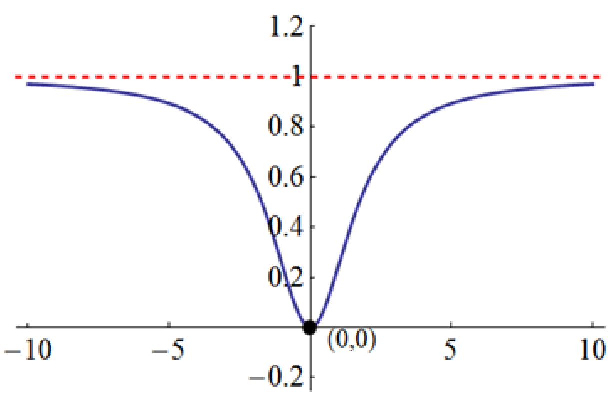
vertical asymptote(s): none
horizontal asymptote(s): $y = 0$
interval(s) of increasing: $[-3, 3]$
interval(s) of decreasing: $(-\infty, -3], [3, \infty)$
local extremum points(s): $x = -3, 3$
interval(s) of concavity: $(-\infty, -3\sqrt{3}], [0, 3\sqrt{3}]$
inflection point(s): $x = -3\sqrt{3}, 0, 3\sqrt{3}$

(f)



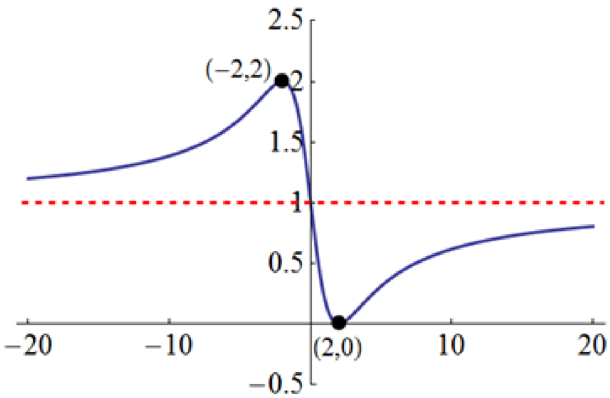
vertical asymptote(s): $x = 1$
horizontal asymptote(s): $y = 0$
interval(s) of increasing: $[-7, 1)$
interval(s) of decreasing: $(-\infty, -7], (1, \infty)$
local extremum points(s): $x = -7$
interval(s) of concavity: $(-\infty, -11]$
inflection point(s): $x = -11$

(g)



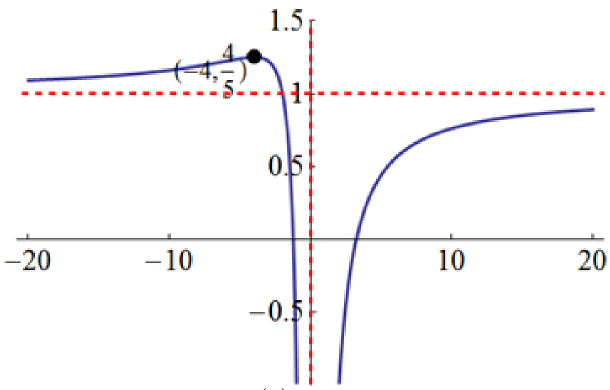
vertical asymptote(s): none
horizontal asymptote(s): $y = 1$
interval(s) of increasing: $[0, \infty)$
interval(s) of decreasing: $(-\infty, 0]$
local extremum points(s): $x = 0$
interval(s) of concavity: $(-\infty, -1], [1, \infty)$
inflection point(s): $x = -1, 1$

(h)



vertical asymptote(s): none
horizontal asymptote(s): $y = 1$
interval(s) of increasing: $(-\infty, -2], [2, \infty)$
interval(s) of decreasing: $[-2, 2]$
local extremum points(s): $x = -2, 2$
interval(s) of concavity: $[-2\sqrt{3}, 0], [2\sqrt{3}, \infty)$
inflection point(s): $x = -2\sqrt{3}, 0, 2\sqrt{3}$

(i)



vertical asymptote(s): $x = 0$
horizontal asymptote(s): $y = 1$
interval(s) of increasing: $(-\infty, -4], (0, \infty)$
interval(s) of decreasing: $[-4, 0)$
local extremum points(s): $x = -4$
interval(s) of concavity: $[-6, 0), (0, \infty)$
inflection point(s): $x = -6$

Chapter 4: Integration

1. (a) $27x - 9x^3 + \frac{9}{5}x^5 - \frac{1}{7}x^7 + C$
   (b) $\frac{625}{3}x^3 - 125x^4 + 30x^5 - \frac{10}{3}x^6 + \frac{1}{7}x^7 + C$
   (c) $\frac{2}{3}x^{\frac{3}{2}} + 2\sqrt{x} + C$
   (d) $4t^2 - \frac{8}{3}t^{\frac{3}{4}} + C$
   (e) $-3\cot x + C$
   (f) $4\sec\theta + C$

2. (a) $-\frac{2}{5}\sqrt{2-5x} + C$
   (b) $\frac{1}{2}e^{2x} - e^x + x + C$
   (c) $-\sqrt{1-x^2} + C$
   (d) $\frac{1}{4}(1+x^3)^{\frac{4}{3}} + C$
   (e) $-\frac{1}{2(1+x^2)} + C$
   (f) $2\tan^{-1}\sqrt{x} + C$
   (g) $\cos\frac{1}{x} + C$
   (h) $-\frac{1}{2}e^{-x^2} + C$
   (i) $\ln(2+e^x) + C$
   (j) $\tan^{-1} e^x + C$
   (k) $-\ln|\cos x| + C$
   (l) $x - \ln(1+e^x) + C$

3. (a) $\frac{14}{3}$
   (b) $\frac{1}{3}$
   (c) $\frac{1}{8}$
   (d) $\frac{1}{2}$
   (e) $\frac{2}{3}$
   (f) $\frac{1}{3}$

4. (a) $\frac{32}{3}$
   (b) $\frac{125}{6}$
   (c) 9
   (d) $\frac{5}{6}$
   (e) $\frac{10}{3}$
   (f) $\frac{9}{2}$

Chapter 5: Further techniques of integration

1. (a) $-\cot\frac{x}{2}+C$
   (b) $\frac{1}{6}\sin^6 x+C$
   (c) $\frac{1}{4}\sin 2x-\frac{1}{16}\sin 8x+C$
   (d) $3\sin\frac{x}{6}+\frac{3}{5}\sin\frac{5x}{6}+C$
   (e) $\sin x-\frac{1}{3}\sin^3 x+C$
   (f) $\frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$
   (g) $-\frac{1}{\sin x}+\frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}+C$
   (h) $-\frac{1}{2}\cos^2 x+\frac{1}{2}\ln(1+\cos^2 x)+C$
   (i) $\frac{\tan^4}{4}-\frac{\tan^2 x}{2}-\ln|\cos x|+C$
   (j) $-8\cot 2x-\frac{8}{3}\cot^3 2x+C$
   (k) $-\frac{1}{8}\cos 4x-\frac{1}{12}\cos 6x+C$
   (l) $\frac{x}{4}+\frac{\sin 2x}{8}+\frac{\sin 4x}{16}+\frac{\sin 6x}{24}+C$

2. (a) $x\ln x-x+C$
   (b) $\frac{x^3}{3}(\ln x-\frac{1}{3})+C$
   (c) $-\frac{1}{x}((\ln x)^2+2\ln x+2)+C$
   (d) $-(x+1)e^{-x}+C$
   (e) $-\frac{e^{-2x}}{4}(2x^2+2x+1)+C$
   (f) $x\sin x+\cos x+C$
   (g) $-\frac{2x^2-1}{4}\cos 2x+\frac{x}{2}\sin 2x+C$
   (h) $x\sin^{-1}x+\sqrt{1-x^2}+C$
   (i) $-\frac{x}{2}+\frac{1+x^2}{2}\tan^{-1}x+C$
   (j) $x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}+C$
   (k) $\frac{x^2}{4}-\frac{x}{4}\sin 2x-\frac{1}{8}\cos 2x+C$
   (l) $\frac{x}{2}(\sin(\ln x)-\cos(\ln x))+C$

3.

4. (a) $x-\tan^{-1}x+C$
   (b) $\frac{x}{\sqrt{1-x^2}}+C$
   (c) $-\sqrt{1-x^2}+\sin^{-1}x+C$
   (d) $\frac{x}{\sqrt{1+x^2}}+C$
   (e) $\frac{9}{2}\sin^{-1}\frac{x}{3}-\frac{x}{2}\sqrt{9-x^2}+C$
   (f) $\ln|x+\sqrt{4+x^2}|+C$

5. (a) $-x+\frac{1}{2}\ln|\frac{1+x}{1-x}|+C$
   (b) $9x-\frac{3}{2}x^2+\frac{1}{3}x^3-27\ln|3+x|+C$
   (c) $x+\ln(1+x^2)+C$
   (d) $\frac{1}{4}\ln|\frac{x-1}{x+3}|+C$
   (e) $\frac{1}{10\sqrt{2}}\ln|\frac{x-\sqrt{2}}{x+\sqrt{2}}|-\frac{1}{5\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}+C$
   (f) $\frac{1}{x+1}+\frac{1}{2}\ln|x^2-1|+C$
   (g) $-\frac{5x-6}{x^2-3x+2}+4\ln|\frac{x-1}{x-2}|+C$
   (h) $\tan^{-1}x+\frac{5}{6}\ln\frac{x^2+1}{x^2+4}+C$
   (i) $\frac{1}{2}\tan^{-1}x+\frac{1}{4}\ln\frac{(x+1)^2}{x^2+1}+C$
   (j) $x^2+2\ln|x+1|+3\ln|x-3|+C$
   (k) $\tan^{-1}x-\frac{1}{x-1}+\ln\frac{x^2+1}{(x-1)^2}+C$
   (l) $\frac{1}{2(x^2+1)}+\ln|x|-\frac{1}{2}\ln(x^2+1)+C$

6. (a) $-\frac{\cos x}{2\sin^2 x}+\frac{1}{2}\ln|\tan\frac{x}{2}|+C$
   (b) $\tan x-\sec x+C$
   (c) $\frac{1}{\cos x}+\frac{1}{3\cos^3 x}+\ln|\tan\frac{x}{2}|+C$

7. (a) $\frac{1}{4}$
   (b) π
   (c) π
   (d) $\frac{2}{3}\ln 2$
   (e) $\frac{4\pi}{3\sqrt{3}}$
   (f) $\frac{2\pi}{3\sqrt{3}}$
   (g) $\frac{\pi}{2}-1$
   (h) $\frac{1}{2}$
   (i) $-\frac{\pi}{2}\ln 2$

8. (a) Convergent
   (b) Convergent
   (c) Divergent
   (d) Divergent
   (e) Convergent
   (f) Divergent

9. (a) $F(x)+C$ where
$$F(x)=\begin{cases} x, & \text{if } x<0 \\ x^2+x, & \text{if } x\ge 0 \end{cases}$$

(b) $F(x)+C$ where
$$F(x)=\begin{cases} 2x^2-x, & \text{if } x<3 \\ \frac{x^3}{3}+x+3, & \text{if } x\ge 3 \end{cases}$$

(c) $F(x)+C$ where
$$F(x)=\begin{cases} -\frac{x^2}{2}, & \text{if } x<0 \\ \frac{x^2}{2}, & \text{if } x\ge 0 \end{cases}$$

(d) $F(x)+C$ where
$$F(x)=\begin{cases} \frac{x^3}{3}-x-\frac{4}{3}, & \text{if } x<-1 \\ x-\frac{x^3}{3}, & \text{if } -1\le x<1 \\ \frac{x^3}{3}-x+\frac{4}{3}, & \text{if } x\ge 1 \end{cases}$$

(e) $F(x)+C$ where
$$F(x)=\begin{cases} \frac{x^3}{3}-\frac{x^2}{2}, & \text{if } x<0 \\ -\frac{x^3}{3}+\frac{x^2}{2}, & \text{if } 0\le x<1 \\ \frac{x^3}{3}-\frac{x^2}{2}+\frac{1}{3}, & \text{if } x\ge 1 \end{cases}$$

(f) $-\frac{3}{140}(9+12x+14x^2)(1-x)^{\frac{4}{3}}+C$

(g) $-\frac{6+25x^3}{1000}(2-5x^3)^{\frac{5}{3}}+C$

(h) $\frac{e^x}{2}(\cos x+\sin x)+C$

(i) $x\tan x+\ln|\cos x|+C$

(j) $\frac{1}{5}\ln|5x+\sqrt{25x^2-4}|+C$

(k) $\frac{x(3x^2+5)}{8(x^2+1)^2}+\frac{3}{8}\tan^{-1}x+C$

(l) $\frac{1}{x^2+2x+2}+\tan^{-1}(x+1)+C$

(m) $\tan^{-1}2x+\frac{2x}{4x^2+1}+C$

(n) $\sqrt{x(4-x)}+4\sin^{-1}\frac{\sqrt{x}}{2}+C$

(o) $-\frac{2\sqrt{4-x^2}}{x}+C$

(p) $-\frac{\cos x}{\cos x+\sin x}+C$

(q) $2\sqrt{1-\sin x}+C$