MATH1010 University Mathematics 2014-2015
Assignment 3
Due: 27 Oct 2013 (Monday)

Answer all questions.

1. Find $\dfrac{dy}{dx}$ for the following implicit functions.
   (a) $x^2y - 5xy^3 = 5$
   (b) $e^{xy} + x^2 \ln y = 2$

2. Find all relative and absolute extrema of the following functions on the given intervals.
   (a) $f(x) = x^4 + 4x^3 - 20x^2 + 80$; $[-6, 3]$
   (b) $f(x) = \dfrac{x-2}{x^2 - 3x + 3}$; $[0, \infty)$
   (c) $f(x) = |x^2 - 16| + 2x$; $[0, 5]$
   (d) $f(x) = \begin{cases} -x^2 - 2x + 4, & \text{if } x \le 1 \\ -x^2 + 6x - 4, & \text{if } x > 1 \end{cases}$; $[-2, 2]$

3. Let $f : [a, b] \to \mathbb{R}$ be a function which is continuous on $[a, b]$ and differentiable on (a, b). Suppose $f'(x) = 0$ for any $x \in (a, b)$. Prove that f is a constant function.

4. Evaluate the following limits.
   (a) $\lim\limits_{x \to 0} \dfrac{x \sin x}{1 - \cos x}$
   (b) $\lim\limits_{x \to 0} \dfrac{\sinh^2 x}{e^x - x - 1}$
   (c) $\lim\limits_{x \to 0} \dfrac{\ln(1 - x^2)}{\ln(\cos x)}$
   (d) $\lim\limits_{x \to 1} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln x} \right)$
   (e) $\lim\limits_{x \to +\infty} \dfrac{\ln(x^4 - 3x + 5)}{\ln(x^3 + 5x^2 - 2)}$
   (f) $\lim\limits_{x \to +\infty} (1 + e^{2x})^{\frac{3}{x}}$

End