HW 2

13.2 3   13.3 13, 15   13.4 5.6   13.5 7

14.1 14 15   14.2 41   14.3 4   14.4 9, 45

13.2 3

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[(\sin t)i + (1+\cos t)j + (\sec^2 t)k\right] dt$$

$$= \left((-\cos t)i + (t+\sin t)j + (\tan t)k\right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= 0 \cdot i + \left(\frac{\pi}{2} + \sqrt{2}\right)j + 2k$$

$$= \left(\frac{\pi}{2} + \sqrt{2}\right)j + 2k$$

13.3 13

$$\vec{r}(t) = (e^t \cos t, e^t \sin t, e^t) \quad -\ln 4 \le t \le 0$$

$$v(t) = \vec{r}'(t) = \left(e^t(-\sin t) + e^t \cos t, e^t(\cos t + \sin t), e^t\right)$$

$$s(t) = \int_0^t |v(\tau)|\, d\tau$$

$$= \int_0^t \left(e^{2\tau}(\cos\tau - \sin\tau)^2 + e^{2\tau}(\cos\tau + \sin\tau)^2 + e^{2\tau}\right)^{\frac{1}{2}} d\tau$$

$$= \int_0^t \sqrt{3} e^\tau d\tau \qquad \text{Lenth} = \int_{-\ln 4}^0 \sqrt{3} e^t dt$$

$$= \sqrt{3}(e^t - 1) \qquad = \frac{3\sqrt{3}}{4}$$

$$t(s) = \ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)$$

$$\vec{r}(s) = \vec{r}(t(s)) = \left(\frac{s+\sqrt{3}}{\sqrt{3}}\cos\left(\ln\frac{s+\sqrt{3}}{\sqrt{3}}\right), \frac{s+\sqrt{3}}{\sqrt{3}}\sin\left(\ln\frac{s+\sqrt{3}}{\sqrt{3}}\right), \frac{s+\sqrt{3}}{\sqrt{3}}\right)$$

15 $\vec{r}(t) = (\sqrt{2}t, \sqrt{2}t, 1-t^2)$

$\vec{r}(0) = (0,0,1)$ $\vec{r}(1) = (\sqrt{2}, \sqrt{2}, 0)$

$\vec{r}'(t) = (\sqrt{2}, \sqrt{2}, -2t)$

Length $= \int_0^1 |\vec{r}'(t)|\, dt = \int_0^1 2\sqrt{1+t^2}\, dt$

$$\int_0^1 \sqrt{1+t^2}\, dt = \int_0^{\frac{\pi}{4}} \sec\theta\, \cancel{\tan\theta}\, d\tan\theta = \int_0^{\frac{\pi}{4}} \sec^3\theta\, d\theta$$

$$= \sec\theta\tan\theta \Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan^2\theta \sec\theta\, d\theta$$

$$= \sqrt{2} - \int_0^{\frac{\pi}{4}} \sec^3\theta\, d\theta + \int_0^{\frac{\pi}{4}} \sec\theta\, d\theta$$

$$\int_0^{\frac{\pi}{4}} \sec^3\theta\, d\theta = \frac{\sqrt{2}}{2} + \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec\theta\, d\theta.$$

$$\int_0^{\frac{\pi}{4}} \sec\theta\, d\theta = \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta}\, d\theta$$

$u = \sec\theta + \tan\theta$

$du = (\sec^2\theta + \tan\theta\sec\theta)\, d\theta$

$$\int_0^{\frac{\pi}{4}} \sec\theta\, d\theta = \int_1^{\sqrt{2}+1} \frac{du}{u} = \ln u \Big|_1^{\sqrt{2}+1} = \ln(\sqrt{2}+1)$$

$$\int_0^{\frac{\pi}{4}} \sec^3\theta\, d\theta = \frac{\sqrt{2}}{2} + \frac{1}{2}\ln(\sqrt{2}+1)$$

Length $= 2\int_0^{\frac{\pi}{4}} \sec^3\theta\, d\theta = \sqrt{2} + \ln(\sqrt{2}+1)$

13.4 5. a.

$\vec{r}(x) = (x, f(x))$ $\vec{r}'(x) = (1, f'(x))$

$$\vec{U}(x) = \frac{\vec{r}'(x)}{|\vec{r}'(x)|} = \frac{1}{\sqrt{1+(f'(x))^2}}(1, f'(x))$$

$$S(x) = \int_0^x \sqrt{1+(f'(\tau))^2}\, d\tau$$

$$\frac{ds}{dx} = \sqrt{1+(f'(x))^2} \qquad \frac{dx}{ds} = \frac{1}{\sqrt{1+(f'(x))^2}}$$

$$\left|\frac{d\vec{v}}{ds}\right| = \left|\frac{dx}{ds}\right|\left|\frac{d\vec{v}}{dx}\right|$$

$$\frac{d\vec{v}}{dx} = \left(\frac{-f'(x)f''(x)}{\left(\sqrt{1+(f'(x))^2}\right)^3}, \frac{f''(x)}{\left(\sqrt{1+(f'(x))^2}\right)^3} \right)$$

$$\left|\frac{d\vec{v}}{dx}\right| = \frac{|f''(x)|}{\left(\sqrt{1+(f'(x))^2}\right)^2} = \frac{|f''(x)|}{(1+(f'(x))^2)}$$

$$K = \left|\frac{d\vec{v}}{ds}\right| = \frac{|f''(x)|}{\left(\sqrt{1+(f'(x))^2}\right)^3}$$

b. $f(x) = \ln(\cos x)$

$$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$$

$$f''(x) = -\sec^2 x$$

$$K(x) = \frac{|-\cancel{\tan}\sec^2 x|}{\left(\sqrt{1+(-\tan x)^2}\right)^3} = \frac{\cos x}{\cancel{\sec x}} \qquad \frac{\pi}{2} < x < \frac{\pi}{2}$$

c. At the point of inflection $f''(x) = 0$

$$K(x) = 0$$

6. a. $\vec{v}(t) = (x'(t), y'(t))$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{1}{\sqrt{(x'(t))^2 + (y'(t))^2}}(x'(t), y'(t))$$

$$\frac{ds}{dt} = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\frac{dt}{ds} = \frac{1}{\sqrt{(x'(t))^2 + (y'(t))^2}}$$

$$k(t) = \left|\frac{d\vec{T}(t)}{ds}\right| = \left|\frac{d\vec{T}(t)}{dt}\right|\left|\frac{dt}{ds}\right|$$

$$\frac{d\vec{T}}{dt} = \left(\frac{\dot{y}(\ddot{x}\dot{y} - \ddot{y}\dot{x})}{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}} , \frac{\dot{x}(\ddot{y}\dot{x} - \ddot{x}\dot{y})}{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}} \right)$$

$$\left|\frac{d\vec{T}}{dt}\right| = \frac{|\ddot{x}\dot{y} - \ddot{y}\dot{x}|}{\dot{x}^2 + \dot{y}^2}$$

$$k(t) = \frac{|\ddot{x}\dot{y} - \ddot{y}\dot{x}|}{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}$$

b. $x(t) = t \quad y(t) = \ln(\sin t)$

$\dot{x}(t) = 1 \quad \dot{y}(t) = \frac{\cos t}{\sin t}$

$\ddot{x}(t) = 0 \quad \ddot{y}(t) = \frac{-1}{\sin^2 t}$

$$k(t) = \frac{\left|\frac{1}{\sin^2 t}\right|}{(1+(\cot t)^2)^{\frac{3}{2}}} = \sin t \quad 0 < t < \pi$$

c. $x(t) = \tan^{-1}(\sinh t) \quad y(t) = \ln \cosh t$

$x'(t) = \frac{\cosh t}{1+(\sinh t)^2} \quad y'(t) = \frac{\sinh t}{\cosh t}$

$x''(t) = -(\operatorname{sech} t)\tanh t \quad y''(t) = (\operatorname{sech} t)^2.$

$x'(t) = \operatorname{sech} t$

$$k(t) = \frac{|(\operatorname{sech} t)^3 + (\operatorname{sech} t)(\tanh t)^2|}{((\operatorname{sech} t)^2 + (\tanh t)^2)^{\frac{3}{2}}}$$

$$= \operatorname{Sech} x$$

13.5 7. $\vec{r}(t) = (\cos t)i + (\sin t)j - k \qquad t = \frac{\pi}{4}$

$\vec{r}(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1)$

$\vec{v}(t) = (-\sin t, \cos t, 0)$

$\vec{T}(t) = \frac{\vec{v}(t)}{|v(t)|} = \frac{(-\sin t, \cos t, 0)}{1} = (-\sin t, \cos t, 0)$

$\vec{T}(\frac{\pi}{4}) = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

$\frac{d\vec{T}}{dt} = (-\cos t, -\sin t, 0)$

$\vec{N}(t) = \frac{d\vec{T}}{dt} / |\frac{d\vec{T}}{dt}| = (-\cos t, -\sin t, 0)$

$\vec{N}(\frac{\pi}{4}) = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0)$

$$\vec{B} = \vec{T} \times \vec{N} = \begin{vmatrix} i & j & k \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \end{vmatrix} = (0, 0, 1)$$

$P(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1)$ lies on the plane

ocsulating plane $\quad \vec{B} = (0, 0, 1)$

$z + 1 = 0 \Rightarrow z = -1$

normal plane $\vec{T} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$$-\frac{\sqrt{2}}{2}\left(x-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}\left(y-\frac{\sqrt{2}}{2}\right) + 0(z+1) = 0$$

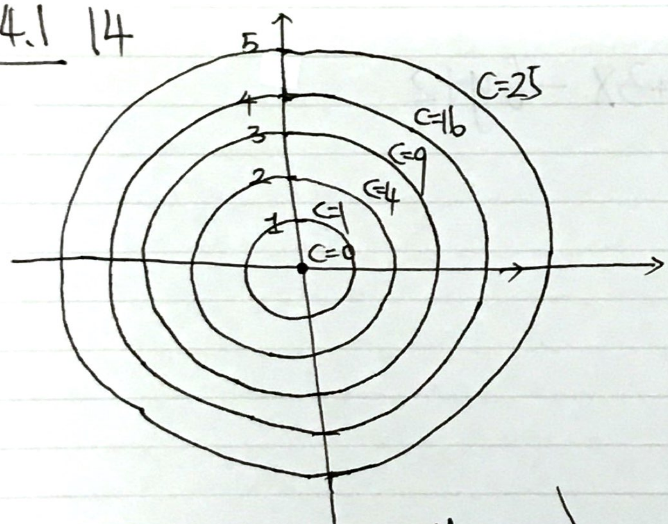
$$\Rightarrow \quad x-y=0$$

Rectifying plane $\vec{N} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)$

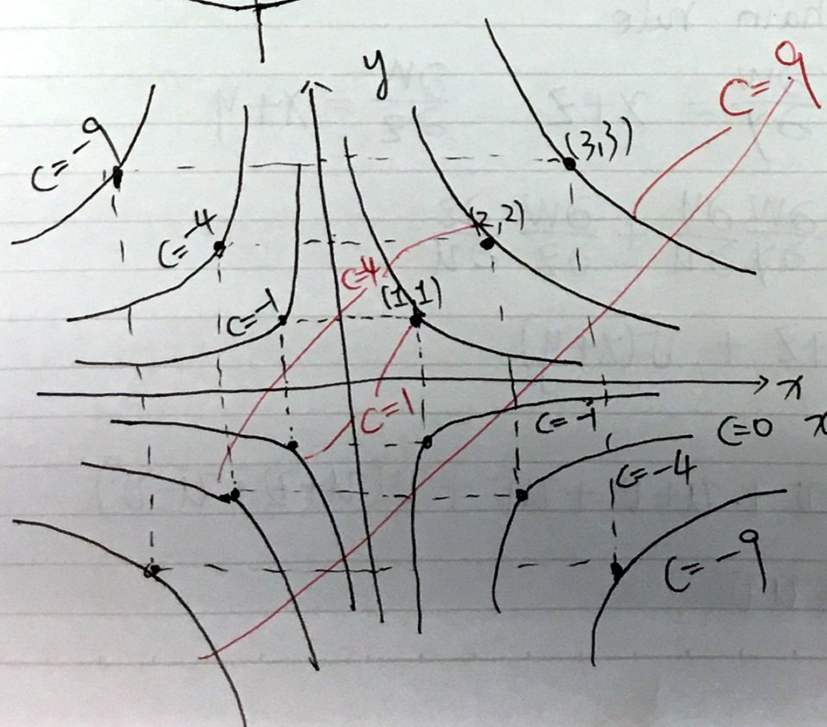
$$-\frac{\sqrt{2}}{2}\left(x-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(y-\frac{\sqrt{2}}{2}\right) + 0(z+1) = 0$$

$$\Rightarrow \quad x+y-\sqrt{2}=0$$

14.1 14



15



C=0 x axis and y axis

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

$$= y+z-(x+z)+u(x+y)$$

$$= -2v+2u^2$$

Express w directly in terms of u and v

$$w = (u+v)(u-v) + (u-v)uv + (u+v)uv$$

$$= u^2-v^2+2u^2v$$

$$\frac{\partial w}{\partial u} = 2u+4uv$$

$$\frac{\partial w}{\partial v} = -2v+2u^2$$

(b) $$\left.\frac{\partial w}{\partial u}\right|_{(u,v)=(\frac{1}{2},1)} = \cancel{+1} \; 3$$

$$\left.\frac{\partial w}{\partial v}\right|_{(u,v)=(\frac{1}{2},1)} = -\frac{3}{2}$$

45. $\frac{\partial u}{\partial x} = x \quad \frac{\partial u}{\partial y} = -y \quad \frac{\partial v}{\partial x} = y \quad \frac{\partial v}{\partial y} = x$

$$w_x = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = xf_u + yf_v$$

$$w_{xx} = f_u + x\left(f_{uu}\frac{\partial u}{\partial x} + f_{uv}\frac{\partial v}{\partial x}\right) + y\left(f_{vu}\frac{\partial u}{\partial x} + f_{vv}\frac{\partial v}{\partial x}\right)$$

$$= f_u + x^2f_{uu} + xyf_{uv} + xyf_{vu} + y^2f_{vv}$$

$$W_y = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = -y f_u + x f_v$$

$$\begin{aligned} W_{yy} = & -f_u + y^2 f_{uu} - xy f_{uv} \\ & + x^2 f_{vv} - xy f_{vu}. \end{aligned}$$

$$W_{xx} + W_{yy} = (x^2 + y^2)(f_{uu} + f_{vv}) = 0$$