MATH4220 PDE Quiz 2 (10 points) April 6, 2017

1. (3 points) Can the eigenvalue problem

$$\begin{aligned} -X''(x) &= \lambda X(x), & 0 < x < 1\\ X(0) &= 0, & X'(1) = 0 \end{aligned}$$

have nonpositive eigenvalues? Prove your statement.

- 2. (3 points) Find the Fourier sine series of f(x) = x on $(0, \pi)$. Then find the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$ by using Parseval's identity.
- 3. (4 points) Solve the following problem

$$\begin{cases} \partial_t u = \partial_x^2 u, & 0 < x < \pi, \quad t > 0 \\ u(0,t) = 0, & u(\pi,t) = 0, \quad t > 0 \\ u(x,t=0) = x, & 0 < x < \pi \end{cases}$$