

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH3070 (Second Term, 2016–2017)
Introduction to Topology
Exercise 0 Preparation (Analysis)

Remarks

These exercises may give you an impression of the foundation needed in this course.

1. Write down precise and concise statements of the following.
 - (a) The definition of the *continuity* of a function $f: X \subset \mathbb{R} \rightarrow \mathbb{R}$ at a point $x_0 \in X$.
 - (b) The definition of the limit of a function $f: X \subset \mathbb{R} \rightarrow \mathbb{R}$ as $x \rightarrow x_0 \in X$.
 - (c) The relation between the continuity of a function $f: X \subset \mathbb{R} \rightarrow \mathbb{R}$ at $x_0 \in X$ and a sequence $x_n \rightarrow x_0 \in X$.
2.
 - (a) The definition of supremum and infimum of a set $A \subset \mathbb{R}$.
 - (b) The definition of supremum and infimum of a subset A in a partially ordered set X .
3. What is a metric on a set X ?
4. What is an ℓ_p -metric on \mathbb{R}^n ? Why is $p \geq 1$ necessary? What is ℓ_∞ -metric?
What is the relation between ℓ_p and ℓ_q metrics?
5. Give three different examples of metric space.
6. Do Question 1 again for X being a subset of a metric space.
7. Given a metric space, how to define an open set (either by using or not using the concept of interior points).