MATH 5070 Exam 1 due *noon, Oct 13, 2014*

Instructions:

- **No** *discussion or internet research. The exam is* **closed book** *except for* **only** *the following references:*
  - *Hirsh, Differential Topology*
  - *Lang, Fundamentals of Differential Geometry*
  - *Notes on Smale's lectures on Differential Topology (on course website)*
  - *Bredon, Topology and Geometry*
- *Cite theorems and definitions from the above references as necessary, but clearly specify the theorem number or page number from these books. You may freely quote theorems on fundamental groups, covering spaces, basic homology theory from [Bredon] if that's helpful.*
- *All manifolds below are assumed to be finite dimensional, Hausdorff and paracompact.*
- *Hand in your exams to Beibei Liu at AB1 #407a. In case you have problem finding her, give it to the secretaries on 6F of AB1.*

1. *Show that the submanifold of* $\mathbb{C}^{n+1}$

$$E = \{(z_0, \cdots, z_n) \in \mathbb{C}^{n+1} | z_0^2 + \cdots + z_n^2 = 1\}$$

is diffeomorphic to the total space of the tangent bundle of the unit sphere S^n.

2. *Show that if* X *is any manifold with boundary, then there exists a smooth nonnegative function* f *on* X *with regular value at 0, such that* $\partial X = f^{-1}(0)$.

Hint: *Use partition of unity.*

3. *Let* M *be a connected manifold and* $X \subset M$ *be a codimension one connected submanifold with nontrivial normal bundle. Show that* $M - X$ *is connected.*

Hint: *Use the tubular neighborhood theorem.*