Date

MATH2230 Complex Variables with Applications

Homework 1

(due on Sept. 12)

Ch1, Sect 3, No. 1, 5

Ch1, Sect 5, No. 5, 6, 8

Ch1, Sect 6, No. 2, 13

Ch1, Sect 11, No. 1, 2, 3

Ch1, SEC. 3 Exercises

1. Reduce each of these quantities to a real number

(a) $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$ (b) $\frac{5i}{(1-i)(2-i)(3-i)}$ (c) $(1-i)^4$

Ans. (a) $-\frac{2}{5}$ (b) $-\frac{1}{2}$ (c) -4

5. Derive expression (6), Sec. 3, for the quotient z_1/z_2 by the method described just after i

Ch1, SEC. 5 Exercises

5. In each case, sketch the set of points determined by the given condition:

(a) $|z-1+i|=1$ (b) $|z+i|\leq 3$ (c) $|z-4i|\geq 4$

6. Using the fact that $|z_1-z_2|$ is the distance between two points z_1 and z_2, give a geometric argument that $|z-1|=|z+i|$ represents the line through the origin whose slope is -1.

8. Let z_1 and z_2 denote any complex numbers

$$z_1=x_1+iy_1 \quad \text{and} \quad z_2=x_2+iy_2$$

Use simple algebra to show that

$$|(x_1+iy_1)(x_2+iy_2)| \quad \text{and} \quad \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$$

are the same and then point out how the identity

$$|z_1z_2|=|z_1|\cdot|z_2|$$

follows.

Ch1, SEC. 6 Exercises

2. Sketch the set of points determined by the condition

(a) $\operatorname{Re}(\bar{z}-i)=2$ (b) $|2\bar{z}+i|=4$

13. Show that the equation $|z - z_0| = R$ of a circle, centered at z_0 with radius R, can be written

$$|z|^2 - 2\mathrm{Re}(z\bar{z}_0) + |z_0|^2 = R^2$$

Ch 1, SEC. 11 Exercises

1. Find the square roots of (a) $2i$; (b) $1 - \sqrt{3}i$ and express them in rectangular coordinates.
   Ans. (a) $\pm(1+i)$; (b) $\pm\frac{\sqrt{3} - i}{\sqrt{2}}$
2. Find the three cube roots c_k $(k=0,1,2)$ of $-8i$, express them in rectangular coordinates, and point out why they are as shown in Fig. 15.

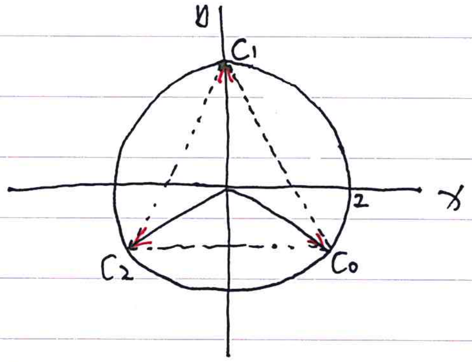

Figure 15

   Ans. $\pm\sqrt{3} - i$, $2i$.
3. Find $(-8 - 8\sqrt{3}i)^{1/4}$, express the roots in rectangular coordinates, exhibit them as the vertices of a certain square, and point out which is the principal root.
   Ans. $\pm(\sqrt{3} - i)$, $\pm(1 + \sqrt{3}i)$